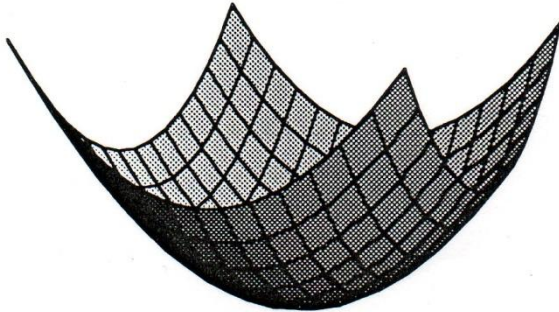
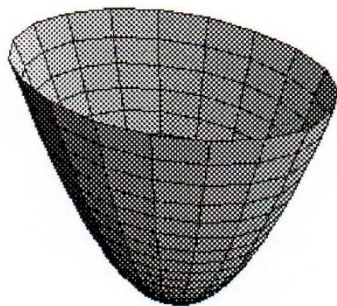


Primjeri ploha u prostoru \mathbb{R}^3



Rotacioni paraboloid

$$z = x^2 + y^2$$



Eliptički paraboloid

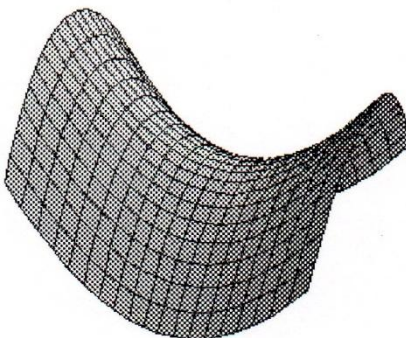
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2pz$$

Parametarske jednađbe

$$x = au \cos t$$

$$y = bu \sin t$$

$$z = \frac{1}{2p} u^2$$



Hiperbolički paraboloid

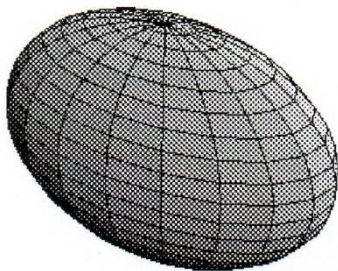
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2pz$$

Parametarske jednađbe

$$x = au \operatorname{ch} t$$

$$y = bu \operatorname{sh} t$$

$$z = \frac{1}{2p} u^2$$



Elipsoid

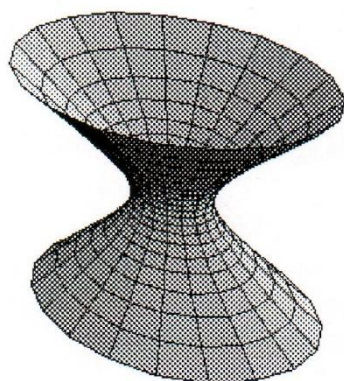
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Parametarske jednađbe

$$x = a \sin u \cos t$$

$$y = b \sin u \sin t$$

$$z = c \cos u$$



Jednoplolni hiperboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

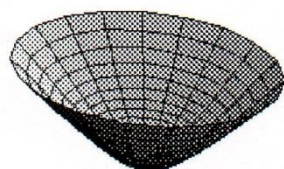
Parametarske jednađbe

$$x = a \operatorname{ch} u \cos t$$

$$y = b \operatorname{ch} u \sin t$$

$$z = c \operatorname{sh} u$$

Jednoplolni hiperboloid je *pravčasta ploha*. To znači da kroz svaku točku na hiperboloidu prolazi pravac koji je čitav u njemu sadržan (štoviše dva takva pravca).



Dvoplolni hiperboloid

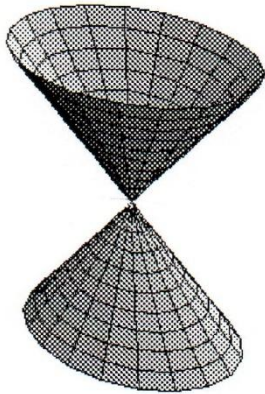
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Parametarske jednađbe

$$x = a \operatorname{sh} u \cos t$$

$$y = b \operatorname{sh} u \sin t$$

$$z = \pm c \operatorname{ch} u$$



Konus drugog reda

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

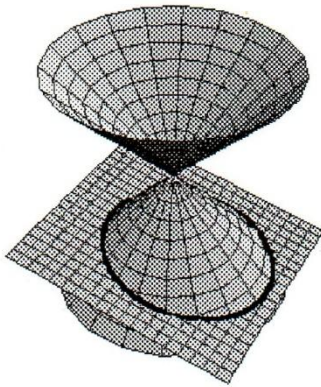
Parametarske jednadžbe

$$x = au \cos t$$

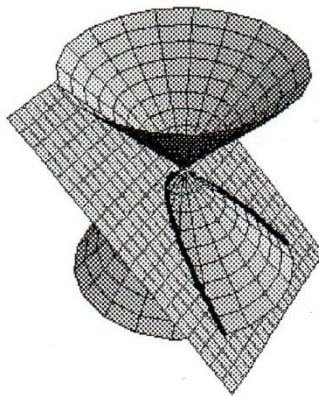
$$y = bu \sin t$$

$$z = cu$$

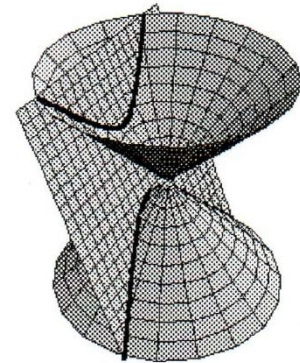
Sijecanjem rotacionog konusa (sa $a=b$) ravninama možemo dobiti sve krivulje drugog reda. Zato se krivulje drugog reda ponekad nazivaju „čunjosječnice“ (konus = čunj).



Elipsa

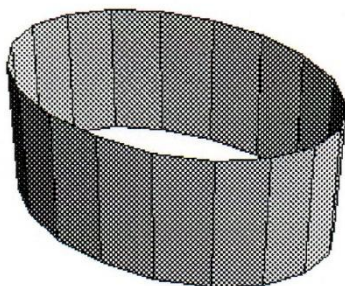


Parabola



Hiperbola

Cilindri drugog reda



Eliptički cilindar

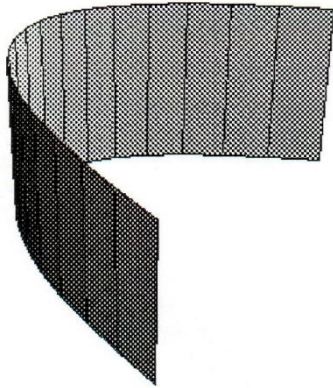
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Parametarske jednadžbe

$$x = a \cos t$$

$$y = b \sin t$$

$$z = u$$



Parabolički cilindar

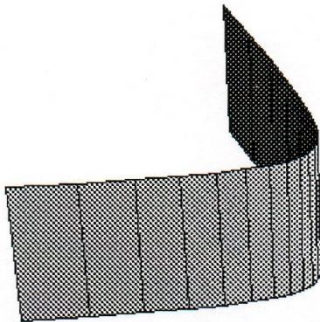
$$y^2 = 2px$$

Parametarske jednađbe

$$x = \frac{1}{2p}t^2$$

$$y = t$$

$$z = u$$



Hiperbolički cilindar

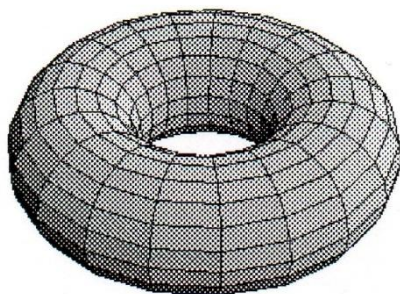
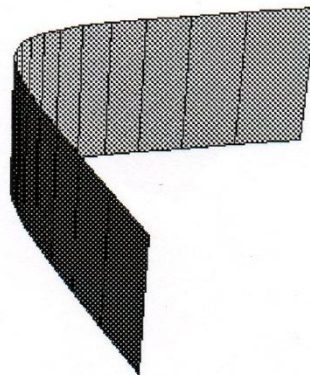
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Parametarske jednađbe

$$x = a \operatorname{ch} t$$

$$y = b \operatorname{sh} t$$

$$z = u$$



Torus

(je algebarska ploha četvrtog reda)

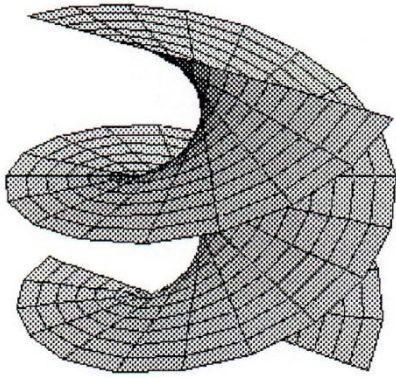
$$(x^2 + y^2 + z^2 - a^2 - b^2)^2 - 4a^2(b^2 - z^2) = 0$$

Parametarske jednađbe

$$x = (a + b \cos u) \cos t$$

$$y = (a + b \cos u) \sin t$$

$$z = b \sin u$$



Helikloid

(je transcendentna ploha, tj. ne može se dobiti kao skup nultočka polinoma u tri varijable)

$$y - x \operatorname{tg} \frac{z}{c} = 0$$

Parametarske jednačbe

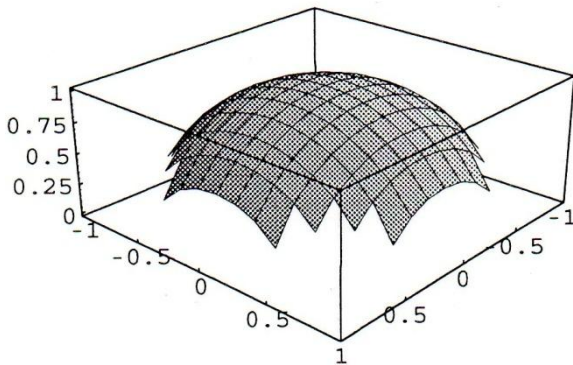
$$x = u \cos t$$

$$y = u \sin t$$

$$z = ct$$

Jos nekoliko primjera ploha

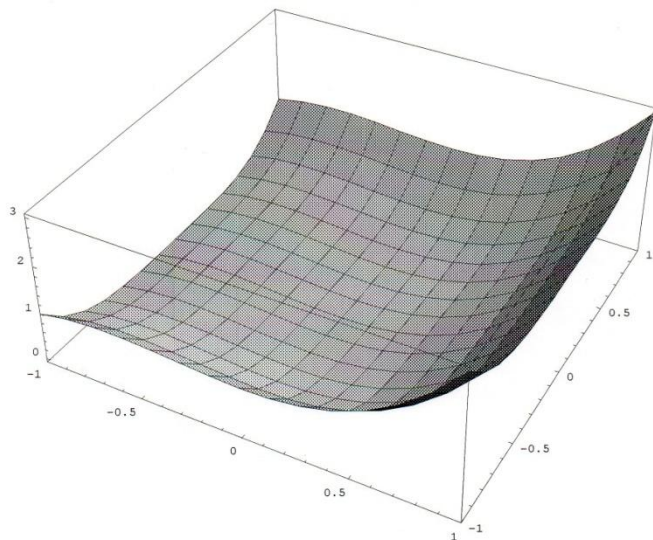
(grafovi dobiveni programom Mathematica)



Graf funkcije:

$$f(x, y) = \frac{1}{\sqrt{1 - x^2 - y^2}}$$

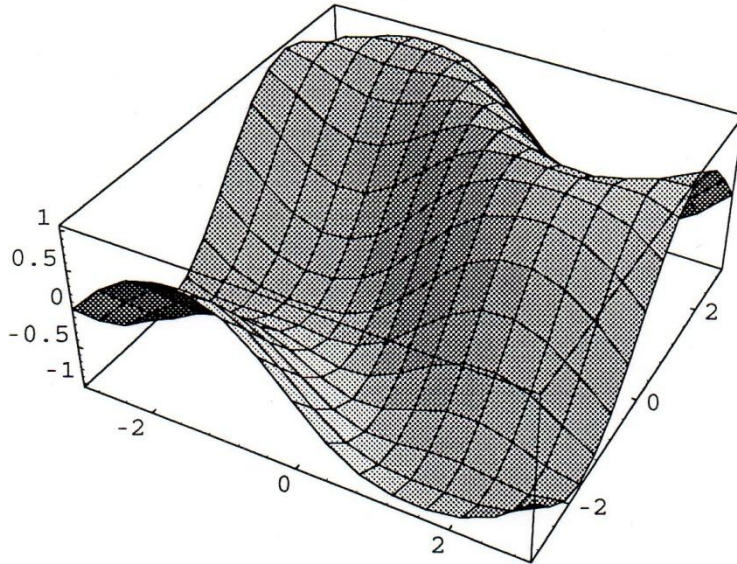
za $(x \in [-1, 1], y \in [-1, 1])$



Graf funkcije:

$$f(x, y) = x^2 + x^3 + y^4$$

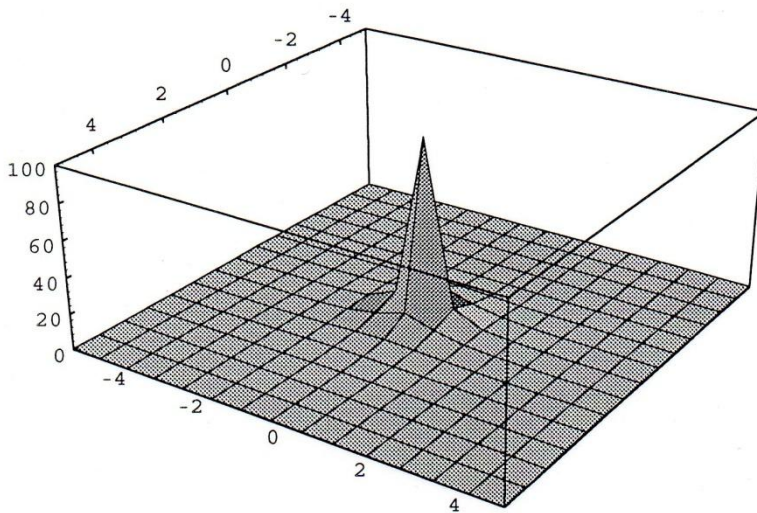
za $(x \in [-1, 1], y \in [-1, 1])$



Graf funkcije:

$$f(x, y) = \sin(y + \sin x)$$

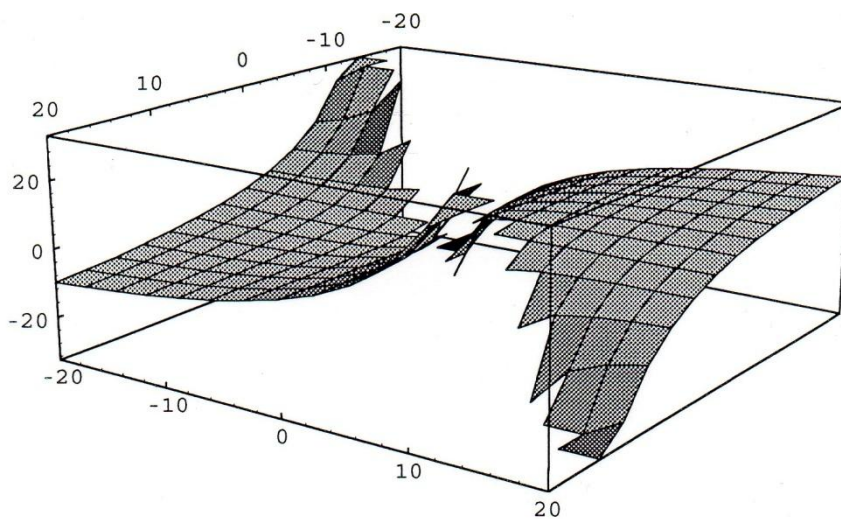
za $(x \in [-3, 3], y \in [-3, 3])$



Graf funkcije:

$$f(x, y) = \frac{5}{x^2 + y^2}$$

za $(x \in [-5, 5], y \in [-5, 5])$



Graf funkcije:

$$f(x, y) = \frac{xy}{x - y}$$

za

$(x \in [-20, 20], y \in [-20, 20])$