Examples of Pseudo-compact Spaces and Their Products

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A space is called pseudo-compact if every map of it to the real line has compact image. Of course this is a fundamental property of compact spaces, but pseudo-compact spaces need not be compact.

The study of such spaces began during the late 1940s, the property having first been defined by J. R. Isbell. During the 1950s and 1960s much effort was given to determining additional properties that such spaces could have. A theory was gradually developed; one of the earliest papers on this subject (1955) was written by S. Mardešić and P. Papić of Zagreb. Many other authors became involved as well.

In general, examples of pseudo-compact spaces that are not compact were constructed on a case-by-case basis. A famous example of such is the first uncountable ordinal space, which is found in most general topology books. Some models of noncompact pseudo-compacta were constructed to show that this property, unlike that of compactness, is not infinitely or even finitely productive. To put it simply, the product of two pseudo-compact spaces need not be pseudo-compact. So the famous Tychonoff theorem does not apply to this class of spaces.

Recently we have discovered noncompact, pseudo-compact spaces that exist "naturally" in profusion. We have constructed some of these spaces in a concrete manner, induced by considering sets of graphs of maps. Among these we have found a large number having the property that pseudo-compactness is *preserved under finite products*. Going further, we have also located major classes of such spaces that can be constructed using certain collections of subsets of Tychonoff cubes, those spaces that are the uncountable product of copies of the unit interval $[0, 1]$. We were able to show that for many of these, pseudo-compactness is *preserved under arbitrary products*. We will discuss such examples in our presentation.