

# Finite Groups, Designs and Codes<sup>1</sup>

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**Abstract.** We will discuss two methods for constructing codes and designs from finite groups (mostly simple finite groups). This is a survey of the collaborative work by the author with J D Key and B Rorigues.

**Keywords.** Designs, codes, simple groups, maximal subgroups, conjugacy classes

We will discuss two methods for constructing codes and designs for finite groups (mostly simple finite groups). In the first method we discuss construction of symmetric 1-designs and binary codes obtained from the primitive permutation representations, that is from the action on the maximal subgroups, of a finite group  $G$ . This method has been applied to several sporadic simple groups, for example in [1], [5], [6], [9], [10], [11] and [12]. The second method introduces a technique from which a large number of non-symmetric 1-designs could be constructed. Let  $G$  be a finite group,  $M$  be a maximal subgroup of  $G$  and  $C_g = [g] = nX$  be the conjugacy class of  $G$  containing  $g$ . We construct  $1 - (v, k, \lambda)$  designs  $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ , where  $\mathcal{P} = nX$  and  $\mathcal{B} = \{(M \cap nX)^y | y \in G\}$ . The parameters  $v, k, \lambda$  and further properties of  $\mathcal{D}$  are determined. We also study codes associated with these designs. We also apply the second method to the groups  $A_7, PSL_2(q)$  and  $J_1$  respectively.

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