

Abstract

Algebraic reconstruction of some difference sets

Kristijan Tabak

Faculty of Electrical Engineering and Computing , Department
of Applied Mathematics, Unska 3, 10000 Zagreb

The main aim of this talk is developing of a method which provides algebraic reconstruction of some difference sets in groups of a type $\langle a, b \mid a^n = b^2 = 1, a^b \in \langle a \rangle \rangle$. Basic tactics is to develop knowledge and describe a structure of known difference set D , and to use that information to generalize on some bigger cases. A heuristic approach is used for $(64, 28, 12)$ difference set in $G = \langle x, y \mid x^{32} = y^2 = 1, x^y = x^{17} \rangle$. It is known that there is a such difference set, and it can be written as

$$D = (1+x+x^2+x^3+x^4+x^9+x^{10}+x^{11}+x^{13}+x^{16}+x^{17}+x^{20}+x^{21}+x^{25}+x^{30}+x^6) + (1+x^{12}+x^{13}+x^{16}+x^{18}+x^{19}+x^{21}+x^{26}+x^{27}+x^{28}+x^{30}+x^6)y.$$

Suppose that $D = \sum_{t=1}^{16} x^{\alpha_t} + \sum_{s=1}^{12} x^{\beta_s} y$, is a $(64, 28, 12)$ difference set in $G = \langle x, y \mid x^{32} = y^2 = 1, x^y = x^{17} \rangle$. Then we will explore sets of a type $A_1 = \sum_{t=1}^{16} \varepsilon^{\alpha_t}$, $B_1 = \sum_{s=1}^{12} \varepsilon^{\beta_s}$, $\varepsilon^{32} = 1$, and as major part of investigation, with notation $A_2 = A_1^{(2)}$, $B_2 = B_1^{(2)}$ we will prove that

1. $|A_2^{(i)} + (-1)^j B_2^{(i)}| = 4$, $i = 0, \dots, 15$, $j = 0, 1$;
2. $|A_1|^2 + |B_1|^2 = 16$,
3. $\overline{A_1} B_1^{(17)} + \overline{B_1} A_1^{(17)} = 0$

Target is to find solution of such system of equation, without knowing difference set D .