Abstract

Perfect codes in the pancake networks

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An independent set D of vertices in a graph is an *efficient dominating set* (or *perfect code*) if each vertex not in D is adjacent to exactly one vertex in D. The efficient dominating sets in Cayley graphs on the symmetric group were investigated in [1] where, in particular, it was shown the existence of efficient dominating sets in the pancake graphs.

The pancake graph $P_n,\ n \geq 3$, is the Cayley graph on the symmetric group Sym_n of $n!$ permutations $\pi = [\pi_1, \pi_2, \ldots, \pi_n]$, where $\pi_i = \pi(i)$ for any $i \in \{1, \ldots, n\}$, with the generating set $PR = \{r_i \in Sym_n, 1 < i \leq n\}$ of all prefix–reversals r_i reversing the order of any substring $[1, i],\ 1 < i \leq n$, of a permutation π when multiplied on the right, i.e., $[\pi_1, \ldots, \pi_i, \pi_{i+1}, \ldots, \pi_n]r_i = [\pi_i, \ldots, \pi_1, \pi_{i+1}, \ldots, \pi_n]$. It is a connected vertex–transitive $(n-1)$-regular graph of order $n!$. The pancake graph is well–known because of the open combinatorial *pancake problem* [2].

The n-dimensional pancake networks based on the pancake graphs $P_n,\ n \geq 3$, are widely used in computer science as models for interconnection networks [3] such that processors are labeled by permutations of length n, and two processors are connected when the label of one is obtained from the other by some prefix–reversal. It is also known that the efficient dominating set are used in broadcasting algorithms for multiple messages on the star and pancake networks [4].

In this paper we give the full characterization of the perfect codes in the pancake networks. In particular, we show that there are exactly n perfect codes $D_k,\ 1 \leq k \leq n$, in the pancake networks $P_n,\ n \geq 3$, presented as $D_k = \{[k\, \pi_2 \ldots \pi_n],\ \pi_j \in \{1, \ldots, n\} \backslash \{k\},\ 2 \leq j \leq n\}$, such that $|D_k| = (n-1)!$. The descriptions of all connections between permutations from D_k for some fixed $k,\ 1 \leq k \leq n$, are also given. Moreover, we show how these perfect codes are applied in solving some combinatorial and graph–theoretical problems on the pancake graphs.

References

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