On p-adic T-numbers

(Poster)

Tomislav Pejković
Department of Mathematics, University of Zagreb
pejkovic@math.hr

For a transcendental number $\xi \in \mathbb{Q}_p$, denote by $w_n(\xi)$ the upper limit of the real numbers w for which there exist infinitely many integer polynomials $P(X)$ of degree at most n satisfying $0 < |P(\xi)|_p \leq \mathrm{H}(P)^{-w-1}$. Also, denote by $w_n^*(\xi)$ the upper limit of the real numbers w for which there exist infinitely many algebraic numbers α in $\mathbb{Q}_p$ of degree at most n satisfying $0 < |\xi - \alpha|_p \leq \mathrm{H}(\alpha)^{-w-1}$. Let $w(\xi) = \limsup_{n\to\infty} \frac{w_n(\xi)}{n}$ and $w^*(\xi) = \limsup_{n\to\infty} \frac{w_n^*(\xi)}{n}$. Mahler used the functions w_n in order to classify transcendental numbers into three classes: S-numbers are those that have $w(\xi) < \infty$, T-numbers are those with $w(\xi) = \infty$ and $w_n(\xi) < \infty$ for any integer $n \geq 1$ and U-numbers have $w(\xi) = \infty$ and $w_n(\xi) = \infty$ for some integer $n \geq 1$. Koksma's classification into S^*-, T^*- and U^*- numbers is achieved in the same way, just using functions w_n^*, w^* in place of w_n, w. These two classifications coincide.

Almost all numbers are S-numbers and U-numbers contain for example Liouville numbers. But, it was only in 1968 that Schmidt proved the existence of T-numbers in $\mathbb{R}$. Schlickewei adapted this result to the p-adic setting. While Schlickewei showed that p-adic T-numbers do exist, his proof only gave numbers ξ such that $w_n(\xi) = w_n^*(\xi)$ for all integers $n \geq 1$. Since for any p-adic transcendental number ξ we have $w_n^*(\xi) \leq w_n(\xi) \leq w_n^*(\xi) + n - 1$, it is natural to ask whether there exist p-adic numbers ξ such that $w_n(\xi) \neq w_n^*(\xi)$ for some integer n and how large can $w_n(\xi) - w_n^*(\xi)$ really be. Although the second question is, as in the more extensively studied real case, far from being resolved, the main result of this work gives a positive answer to the first question and goes some way in answering the second one.

Theorem. Let $(w_n)_{n\geq 1}$ and $(w_n^*)_{n\geq 1}$ be two non-decreasing sequences in $[1, +\infty]$ such that

$$w_n^* \leq w_n \leq w_n^* + (n-1)/n, \quad w_n > n^3 + 2n^2 + 5n + 2, \qquad \text{for any } n \geq 1.$$

Then there exists a p-adic transcendental number ξ such that

$$w_n^*(\xi) = w_n^* \quad \text{and} \quad w_n(\xi) = w_n, \qquad \text{for any } n \geq 1.$$

We also impose much milder growth requirements on the sequence $(w_n)_{n\geq 1}$ than Schlickewei and thus our theorem considerably improves the range of attainable values for w_n^* and w_n.

MSC2010: 11J04.

Keywords: p-adic numbers, T-numbers.

Section: 3. Number Theory.

.