Tangential Center Problem for Darboux Centers

(Talk)

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One of two classical problems for polynomial vector fields in the plane is the *center problem*. It asks for the determination of all mechanisms leading to a system with a center (singularity bounded by a continuous family of periodic solutions). The problem is solved only for quadratic vector fields.

The problem has its *infinitesimal version*. Here one studies perturbations of a system with a center and asks for the determination of those deformations under which a center survives. *Tangential center problem* asks for deformations preserving the center to the first order. For deformations of Hamiltonian centers, the tangential center problem leads to the problem of identical vanishing of Abelian integrals. Under generic hypothesis on the Hamiltonian, the problem was solved by Il'yashenko in 1969.

In this lecture, I will present the results of a joint work with Colin Christopher. We study deformations of polynomial systems with a center having a Darboux first integral i.e. a first integral of the form $F(x,y) = \Pi_{i=0}^{k} f_i(x,y)^{\alpha_i}$, with $f_i(x,y)$ polynomial and $\alpha_i > 0$. We solve the tangential center problem for such deformations under generic hypothesis on F. The approach is geometric based on complexification of the problem and the study of its monodromy.

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